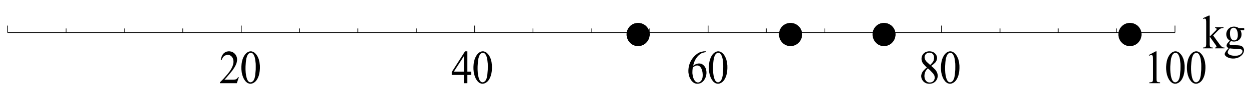
INTRODUCTION TO VECTORS

WHAT IS A VECTOR?

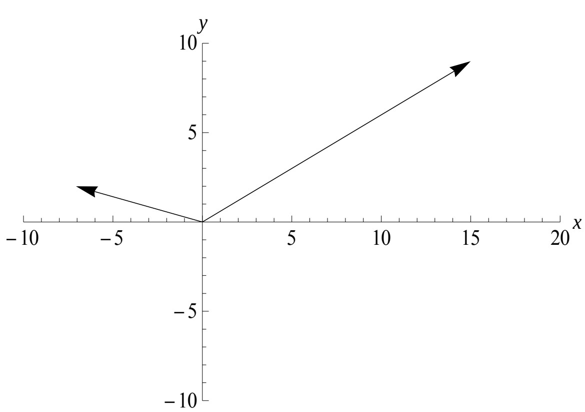
A *scalar* is a number, it has only *magnitude*. Examples of scalars are mass, body temperature, and the volume of a bath tub. A *vector* has both *magnitude* and *direction*. Examples of vectors are velocity, force, magnetic field lines etc.

You can also think of it this way: Scalars have only one dimension, vectors have two or three.

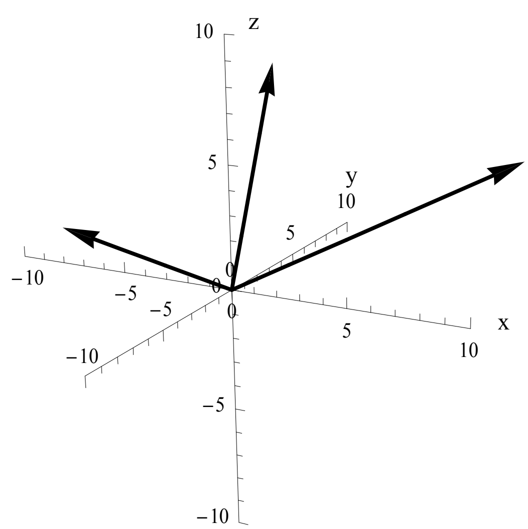
Scalars:



Vectors in 2 dimensions:



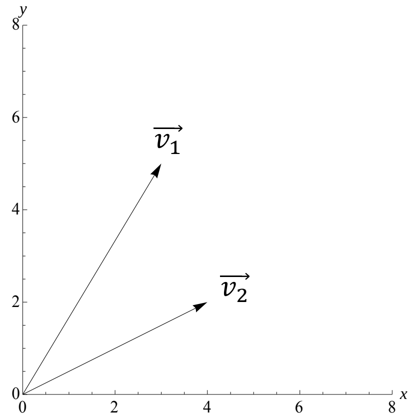
Vectors in 3 dimensions:



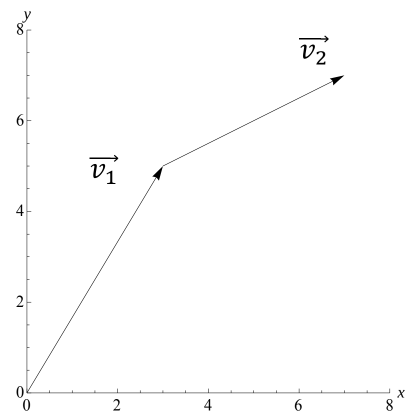
WHAT CAN WE DO WITH VECTORS?

1)ADD TWO VECTORS

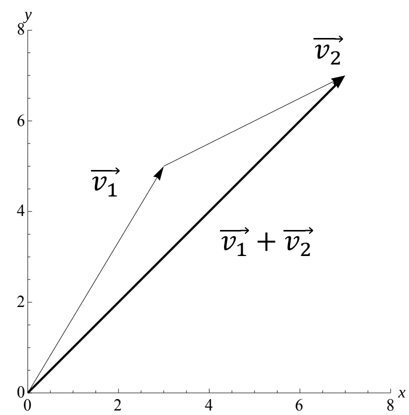
a) "Slap on" method



1. Take 2 vectors

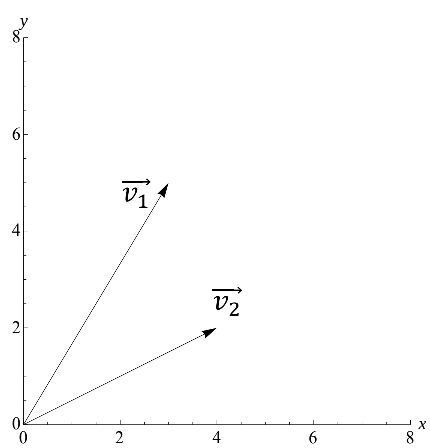


2. "Slap" one at the end of the other

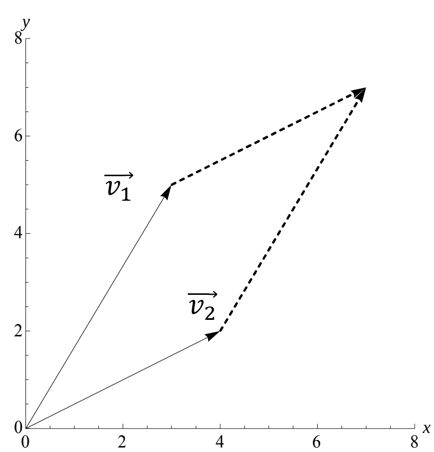


3. Draw out the result

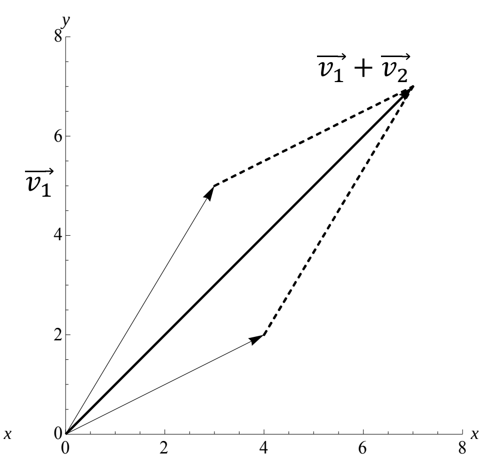
b) Parallelogram method



1. Take 2 vector

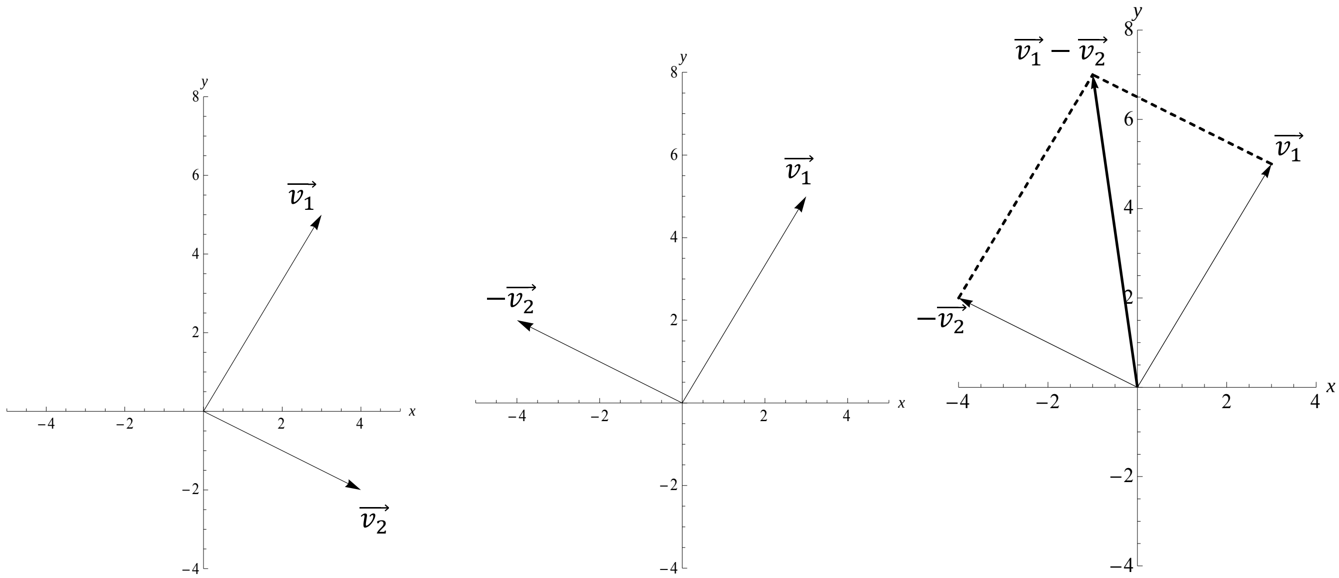


2. Draw parallels to both vectors



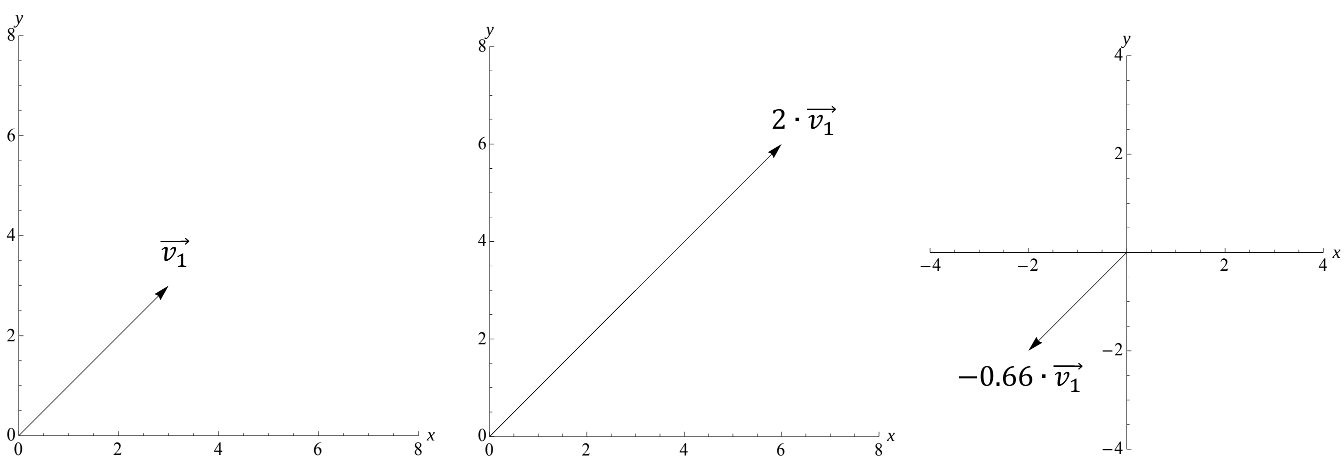
3. The resulting vector is the diagonal of the parallelogram

2)SUBTRACT TWO VECTORS



1. Take 2 vectors 2. "Flip" the one you are subtracting 3. Add the resulting vectors

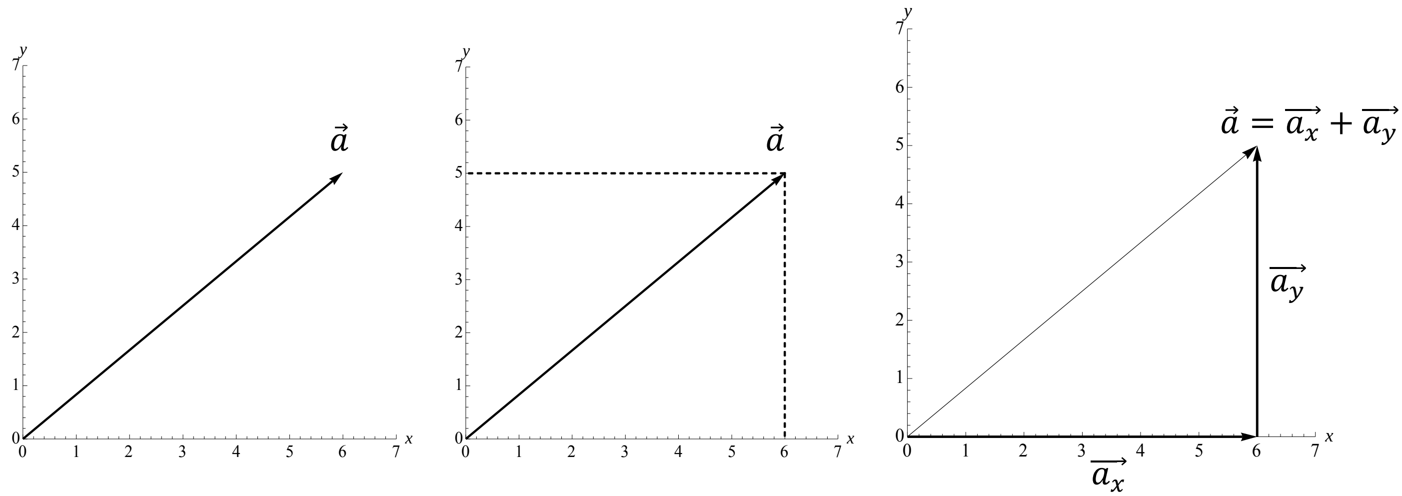
3)MULTIPLY A VECTOR BY A SCALAR



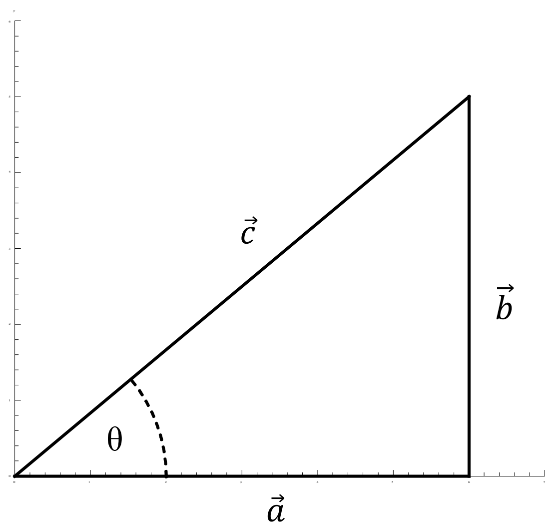
To multiply a vector by a scalar (i.e. a number) you multiply the ***magnitude*** by the number and leave the ***direction*** unchanged, unless you are multiplying by a negative number, in which case the direction "flips" to the opposite side.

HOW TO ADD TWO VECTORS WITHOUT DRAWING A GRAPH?

What you need to do is ***decompose*** each vector into its *x* and *y* components. Notice that the vector $\vec{a}$ is the same as $\overrightarrow{a_x} + \overrightarrow{a_y}$:



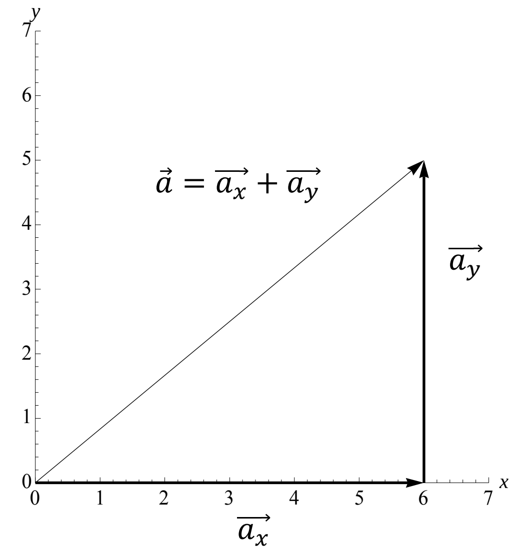
How do we get this decomposition?
We have to use the sine and cosine functions. Recollect that these are defined as:



$$\sin(\theta) = \frac{opposite\ side}{hypothenuse} = \frac{b}{c}$$

$$\cos(\theta) = \frac{adjecent\ side}{hypothenuse} = \frac{a}{c}$$

What does this mean to our vectors?? Take a look:



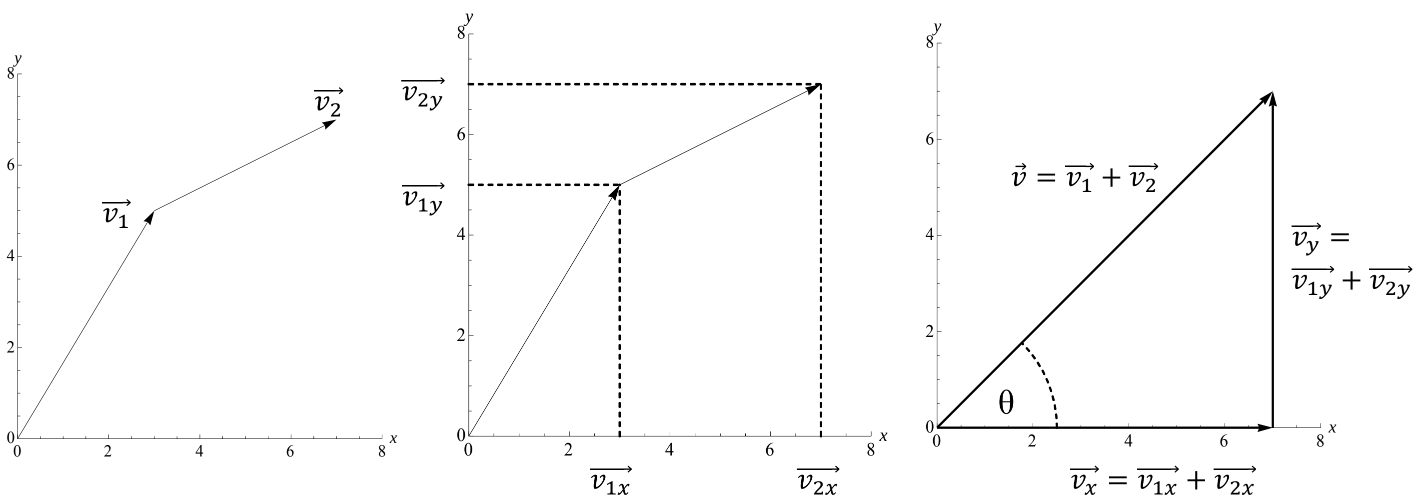
$$\text{Cos}(\theta) = \frac{a_x}{a} \quad / \cdot a$$

$$\boxed{a_x = a \cdot \cos(\theta)}$$

$$\text{Sin}(\theta) = \frac{a_y}{a} \quad / \cdot a$$

$$\boxed{a_y = a \cdot \sin(\theta)}$$

So, how do we add the two vectors? We have to decompose each of the vectors we are adding and then add their x and y components separately.



We get the magnitude of the resulting vector by using the Pythagoras theorem:

$$v^2 = v_x^2 + v_y^2$$

$$\boxed{v = \sqrt{v_x^2 + v_y^2}}$$

And the angle by using the tangent function:

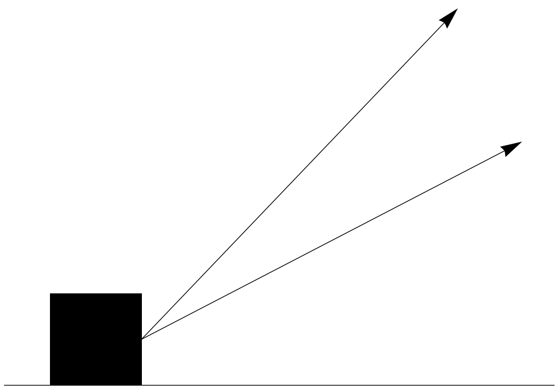
$$\tan(\theta) = \frac{\textit{opposite side}}{\textit{adjecent side}} = \frac{v_y}{v_x} \quad \rightarrow \qquad \boxed{\theta = arctan\left(\frac{v_y}{v_x}\right)}$$

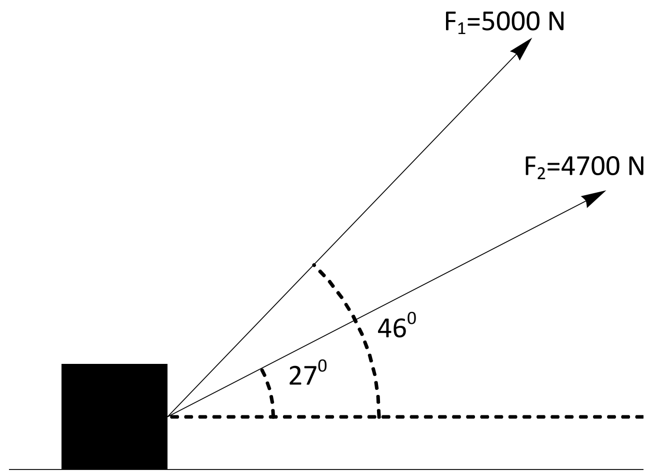
WHAT ARE VECTORS USED FOR?

Consider this problem:

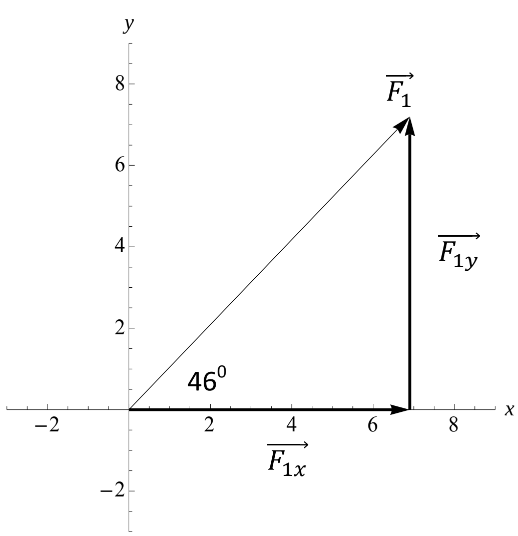
The Egyptians are building a pyramid. They have tied two ropes to a huge block of rock and are pulling on them. The first rope is at an angle of 46^0 and is exerting a force of 5000 Newton; the second at an angle of 27^0 and exerting a force of 4700 Newton. What will the total force on the block be and which way will it be directed?

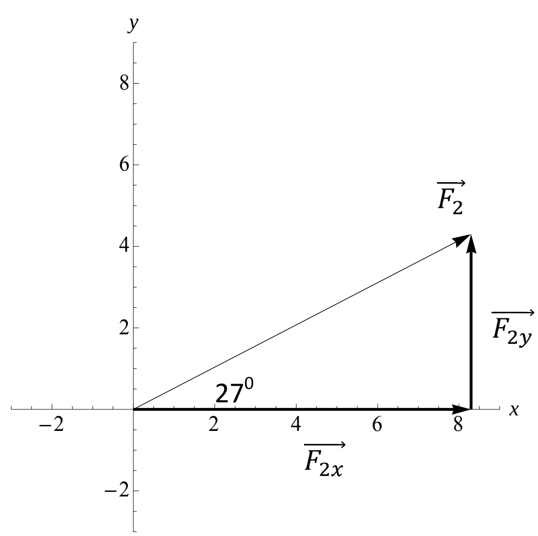
First, lets sketch this situation:

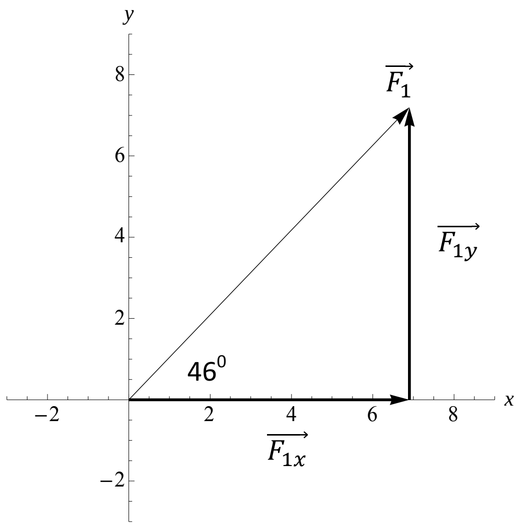


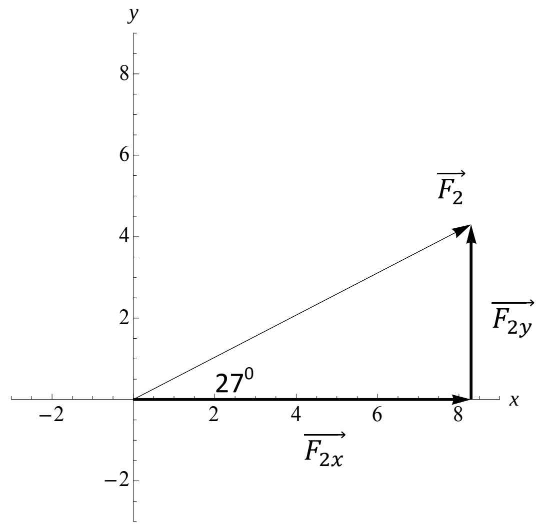


Let us now *decompose* these two vectors into their *x* and *y* components:









$$F_{1x} = F_1 \cdot \cos(46^0) = 5000 \cdot \cos(46^0) = 3473\ N$$

$$F_{1y} = F_1 \cdot \sin(46^0) = 5000 \cdot \sin(46^0) = 3597\ N$$

$$F_{2x} = F_2 \cdot \cos(27^0) = 4700 \cdot \cos(27^0) = 4188\ N$$

$$F_{2y} = F_2 \cdot \sin(27^0) = 4700 \cdot \sin(27^0) = 2134\ N$$

$$F_x = F_{1x} + F_{2x} = 3473\ N + 4188\ N = 7661\ N$$

$$F_y = F_{1y} + F_{2y} = 3597\ N + 2134\ N = 5731\ N$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{7661^2 + 5731^2} = 9567\ N$$

$$\theta = arctan\left(\frac{F_y}{F_x}\right) = \arctan\left(\frac{5731}{7661}\right) = 37^0$$

So, finally, the answer to the problem is:

The total force acting on the block is 9567 Newton and it is directed at an angle of 37^0 to the ground.