12.5 Momentum and Impulse

Momentum is a physical quantity which is closely related to forces. Momentum is a property which applies to moving objects.

Definition: Momentum
Momentum is the tendency of an object to continue to move in its direction of travel. Momentum is calculated from the product of the mass and velocity of an object.

The momentum (symbol p) of an object of mass m moving at velocity v is:

$$p = m \cdot v$$

According to this equation, momentum is related to both the mass and velocity of an object. A small car travelling at the same velocity as a big truck will have a smaller momentum than the truck. The smaller the mass, the smaller the velocity.
A car travelling at 120 km·hr^{-1}will have a bigger momentum than the same car travelling at 60 km·hr^{-1}. Momentum is also related to velocity; the smaller the velocity, the smaller the momentum.
Different objects can also have the same momentum, for example a car travelling slowly can have the same momentum as a motor cycle travelling relatively fast. We can easily demonstrate this. Consider a car of mass 1 000 kg with a velocity of 8 m·s^{-1}(about 30 km·hr^{-1}). The momentum of the car is therefore

$$\begin{aligned} p &= m \cdot v \\ &= (1000\,\mathrm{kg})(8\,\mathrm{m}\cdot\mathrm{s}^{-1}) \\ &= 8000\,\mathrm{kg}\cdot\,\mathrm{m}\cdot\mathrm{s}^{-1} \end{aligned}$$

Now consider a motor cycle of mass 250 kg travelling at 32 m·s^{-1} (about 115 km·hr^{-1}). The momentum of the motor cycle is:

$$\begin{aligned} p &= m \cdot v \\ &= (250\,\mathrm{kg})(32\,\mathrm{m}\cdot\mathrm{s}^{-1}) \\ &= 8000\,\mathrm{kg}\cdot\,\mathrm{m}\cdot\mathrm{s}^{-1} \end{aligned}$$

Even though the motor cycle is considerably lighter than the car, the fact that the motor cycle is travelling much faster than the car means that the momentum of both vehicles is the same.

From the calculations above, you are able to derive the unit for momentum as kg·m·s^{-1}.
Momentum is also vector quantity, because it is the product of a scalar (m) with a vector (v). This means that whenever we calculate the momentum of an object, we need to include the direction of the momentum.

Worked Example 91: Momentum of a Soccer Ball
Question: A soccer ball of mass 420 g is kicked at 20 m·s^{-1} towards the goal post. Calculate the momentum of the ball.
Answer
Step 1 : Identify what information is given and what is asked for
The question explicitly gives

- the mass of the ball, and
- the velocity of the ball

The mass of the ball must be converted to SI units.

$$420 \text{ g} = 0{,}42\,\text{kg}$$

We are asked to calculate the momentum of the ball. From the definition of momentum,

$$p = m \cdot v$$

we see that we need the mass and velocity of the ball, which we are given.

Step 2 : Do the calculation

We calculate the magnitude of the momentum of the ball,

$$\begin{aligned} p &= m \cdot v \\ &= (0{,}42\,\text{kg})(20\,\text{m} \cdot \text{s}^{-1}) \\ &= 8{,}4\,\text{kg} \cdot \text{m} \cdot \text{s}^{-1} \end{aligned}$$

Step 3 : Quote the final answer

We quote the answer with the direction of motion included, p = 8,4 kg·m·s^{-1} in the direction of the goal post.

Worked Example 92: Momentum of a cricket ball

Question: A cricket ball of mass 160 g is bowled at 40 m·s^{-1} towards a batsman. Calculate the momentum of the cricket ball.

Answer

Step 1 : Identify what information is given and what is asked for

The question explicitly gives

- the mass of the ball (m = 160 g = 0,16 kg), and
- the velocity of the ball (v = 40 m·s^{-1})

To calculate the momentum we will use

$$p = m \cdot v$$

.

Step 2 : Do the calculation

$$\begin{aligned} p &= m \cdot v \\ &= (0{,}16\,\text{kg})(40\,\text{m} \cdot \text{s}^{-1}) \\ &= 6{,}4\,\text{kg} \cdot \text{m} \cdot \text{s}^{-1} \\ &= 6{,}4\,\text{kg} \cdot \text{m} \cdot \text{s}^{-1}\text{in the direction of the batsman} \end{aligned}$$

Worked Example 93: Momentum of the Moon

Question: The Moon is 384 400 km away from the Earth and orbits the Earth in 27,3 days. If the Moon has a mass of 7,35 x 10^{22}kg, what is the magnitude of its momentum if we assume a circular orbit?

Answer

Step 1 : Identify what information is given and what is asked for

The question explicitly gives

- the mass of the Moon (m = 7,35 x 10^{22} kg)
- the distance to the Moon (384 400 km = 384 400 000 m = 3,844 x 10^8 m)
- the time for one orbit of the Moon (27,3 days = 27,3 x 24 x 60 x 60 = 2,36 x 10^6 s)

We are asked to calculate only the magnitude of the momentum of the Moon (i.e. we do not need to specify a direction). In order to do this we require the mass and the magnitude of the velocity of the Moon, since

$$p = m \cdot v$$

Step 2 : Find the magnitude of the velocity of the Moon
The magnitude of the average velocity is the same as the speed. Therefore:

$$s = \frac{d}{\Delta t}$$

We are given the time the Moon takes for one orbit but not how far it travels in that time. However, we can work this out from the distance to the Moon and the fact that the Moon has a circular orbit. Using the equation for the circumference, C, of a circle in terms of its radius, we can determine the distance travelled by the Moon in one orbit:

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(3{,}844 \times 10^8 \mathrm{m}) \\ &= 2{,}42 \times 10^9 \ \mathrm{m} \end{aligned}$$

Combining the distance travelled by the Moon in an orbit and the time taken by the Moon to complete one orbit, we can determine the magnitude of the Moon's velocity or speed,

$$\begin{aligned} s &= \frac{d}{\Delta t} \\ &= \frac{C}{T} \\ &= \frac{2{,}42 \times 10^9 m}{2{,}36 \times 10^6 s} \\ &= 1{,}02 \times 10^3 \, \mathrm{m \cdot s^{-1}}. \end{aligned}$$

Step 3 : Finally calculate the momentum and quote the answer
The magnitude of the Moon's momentum is:

$$\begin{aligned} p &= m \cdot v \\ &= (7{,}35 \times 10^{22} \, \mathrm{kg})(1{,}02 \times 10^3 \, \mathrm{m \cdot s^{-1}}) \\ &= 7{,}50 \times 10^{25} \, \mathrm{kg \cdot \ m \cdot s^{-1}} \end{aligned}$$

12.5.1 Vector Nature of Momentum

As we have said, momentum is a vector quantity. Since momentum is a vector, the techniques of vector addition discussed in Chapter 11 must be used to calculate the total momentum of a system.

Worked Example 94: Calculating the Total Momentum of a System

Question: Two billiard balls roll towards each other. They each have a mass of 0,3 kg. Ball 1 is moving at $v_1 = 1\,\mathrm{m\cdot s^{-1}}$ to the right, while ball 2 is moving at $v_2 = 0{,}8\,\mathrm{m\cdot s^{-1}}$ to the left. Calculate the total momentum of the system.

Answer

Step 1 : Identify what information is given and what is asked for

The question explicitly gives

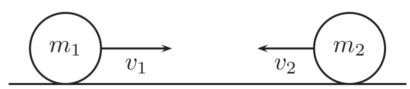
- the mass of each ball,
- the velocity of ball 1, v_1, and
- the velocity of ball 2, v_2,

all in the correct units!

We are asked to calculate the **total momentum of the system**. In this example our system consists of two balls. To find the total momentum we must determine the momentum of each ball and add them.

$$p_{total} = p_1 + p_2$$

Since ball 1 is moving to the right, its momentum is in this direction, while the second ball's momentum is directed towards the left.



Thus, we are required to find the sum of two vectors acting along the same straight line. The algebraic method of vector addition introduced in Chapter 11 can thus be used.

Step 2 : Choose a frame of reference

Let us choose right as the positive direction, then obviously left is negative.

Step 3 : Calculate the momentum

The total momentum of the system is then the sum of the two momenta taking the directions of the velocities into account. Ball 1 is travelling at 1 m·s^{-1}to the right or +1 m·s^{-1}. Ball 2 is travelling at 0,8 m·s^{-1}to the left or -0,8 m·s^{-1}. Thus,

$$\begin{aligned}
p_{total} &= m_1v_1 + m_2v_2 \\
&= (0{,}3\,\mathrm{kg})(+1\,\mathrm{m\cdot s^{-1}}) + (0{,}3\,\mathrm{kg})(-0{,}8\,\mathrm{m\cdot s^{-1}}) \\
&= (+0{,}3\,\mathrm{kg\cdot\ m\cdot s^{-1}}) + (-0{,}24\,\mathrm{kg\cdot\ m\cdot s^{-1}}) \\
&= +0{,}06\,\mathrm{kg\cdot\ m\cdot s^{-1}} \\
&= 0{,}06\,\mathrm{kg\cdot\ m\cdot s^{-1}}\text{to the right}
\end{aligned}$$

In the last step the direction was added in words. Since the result in the second last line is positive, the total momentum of the system is in the positive direction (i.e. to the right).

12.5.2 Exercise

1. (a) The fastest recorded delivery for a cricket ball is 161,3 km·hr^{-1}, bowled by Shoaib Akhtar of Pakistan during a match against England in the 2003 Cricket World Cup, held in South Africa. Calculate the ball's momentum if it has a mass of 160 g.

(b) The fastest tennis service by a man is 246,2 km·hr^{-1}by Andy Roddick of the United States of America during a match in London in 2004. Calculate the ball's momentum if it has a mass of 58 g.

(c) The fastest server in the women's game is Venus Williams of the United States of America, who recorded a serve of 205 km·hr^{-1}during a match in Switzerland in 1998. Calculate the ball's momentum if it has a mass of 58 g.

(d) If you had a choice of facing Shoaib, Andy or Venus and didn't want to get hurt, who would you choose based on the momentum of each ball.

2. Two golf balls roll towards each other. They each have a mass of 100 g. Ball 1 is moving at $v_1 =$ 2,4 m·s^{-1}to the right, while ball 2 is moving at $v_2 =$ 3 m·s^{-1}to the left. Calculate the total momentum of the system.

3. Two motor cycles are involved in a head on collision. Motorcycle A has a mass of 200 kg and was travelling at 120 km·hr^{-1}south. Motor cycle B has a mass of 250 kg and was travelling north at 100 km·hr^{-1}. A and B are about to collide. Calculate the momentum of the system before the collision takes place.

12.5.3 Change in Momentum

Let us consider a tennis ball (mass = 0,1 kg) that is dropped at an initial velocity of 5 m·s^{-1}and bounces back at a final velocity of 3 m·s^{-1}. As the ball approaches the floor it has a momentum that we call the momentum before the collision. When it moves away from the floor it has a different momentum called the momentum after the collision. The bounce on the floor can be thought of as a collision taking place where the floor exerts a force on the tennis ball to change its momentum.

The momentum before the bounce can be calculated as follows:
Because momentum and velocity are vectors, we have to choose a direction as positive. For this example we choose the initial direction of motion as positive, in other words, downwards is positive.

$$\begin{aligned} p_i &= m \cdot v_i \\ &= (0{,}1\,\mathrm{kg})(+5\,\mathrm{m\cdot s^{-1}}) \\ &= 0{,}5\ \ \mathrm{kg\cdot\ m\cdot s^{-1}}\text{downwards} \end{aligned}$$

When the tennis ball bounces back it changes direction. The final velocity will thus have a negative value. The momentum after the bounce can be calculated as follows:

$$\begin{aligned} p_f &= m \cdot v_f \\ &= (0{,}1\,\mathrm{kg})(-3\,\mathrm{m\cdot s^{-1}}) \\ &= -0{,}3\,\mathrm{kg\cdot\ m\cdot s^{-1}} \\ &= 0{,}3\ \ \mathrm{kg\cdot\ m\cdot s^{-1}}\text{upwards} \end{aligned}$$

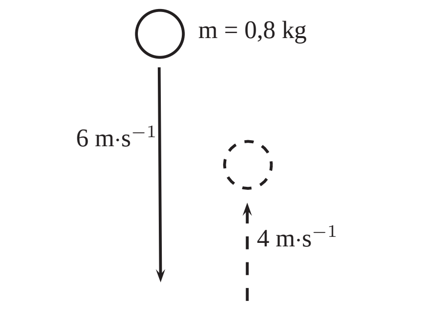
Now let us look at what happens to the momentum of the tennis ball. The momentum changes during this bounce. We can calculate the change in momentum as follows:
Again we have to choose a direction as positive and we will stick to our initial choice as downwards is positive. This means that the final momentum will have a negative number.

$$\begin{aligned} \Delta p &= p_f - p_i \\ &= m \cdot v_f - m \cdot v_i \\ &= (-0{,}3\,\mathrm{kg}) - (0{,}5\,\mathrm{m\cdot s^{-1}}) \\ &= -0{,}8\,\mathrm{kg\cdot\ m\cdot s^{-1}} \\ &= 0{,}8\ \ \mathrm{kg\cdot\ m\cdot s^{-1}}\text{upwards} \end{aligned}$$

You will notice that this number is bigger than the previous momenta calculated. This is should be the case as the ball needed to be stopped and then given momentum to bounce back.

Worked Example 95: Change in Momentum

Question: A rubber ball of mass 0,8 kg is dropped and strikes the floor with an initial velocity of 6 m·s^{-1}. It bounces back with a final velocity of 4 m·s^{-1}. Calculate the change in the momentum of the rubber ball caused by the floor.



Answer

Step 1 : Identify the information given and what is asked

The question explicitly gives

- the ball's mass (m = 0,8 kg),
- the ball's initial velocity (v_i = 6 m·s^{-1}), and
- the ball's final velocity (v_f = 4 m·s^{-1})

all in the correct units.

We are asked to calculate the change in momentum of the ball,

$$\Delta p = mv_f - mv_i$$

We have everything we need to find Δp. Since the initial momentum is directed downwards and the final momentum is in the upward direction, we can use the algebraic method of subtraction discussed in the vectors chapter.

Step 2 : Choose a frame of reference

Let us choose down as the positive direction.

Step 3 : Do the calculation and quote the answer

$$\begin{aligned}
\Delta p &= mv_f - mv_i \\
&= (0{,}8\,\mathrm{kg})(-4\,\mathrm{m\cdot s^{-1}}) - (0{,}8\,\mathrm{kg})(+6\,\mathrm{m\cdot s^{-1}}) \\
&= (-3{,}2\,\mathrm{kg\cdot\ m\cdot s^{-1}}) - (4{,}8\,\mathrm{kg\cdot\ m\cdot s^{-1}}) \\
&= -8 \\
&= 8\ \ \mathrm{kg\cdot m\cdot s^{-1}}\ \text{upwards}
\end{aligned}$$

12.5.4 Exercise

1. Which expression accurately describes the change of momentum of an object?

   A $\frac{F}{m}$

   B $\frac{F}{t}$

   C $F \cdot m$

   D $F \cdot t$

2. A child drops a ball of mass 100 g. The ball strikes the ground with a velocity of 5 m·s^{-1}and rebounds with a velocity of 4 m·s^{-1}. Calculate the change of momentum of the ball.

3. A 700 kg truck is travelling north at a velocity of 40 km·hr^{-1}when it is approached by a 500 kg car travelling south at a velocity of 100 km·hr^{-1}. Calculate the total momentum of the system.

12.5.5 Newton's Second Law revisited

You have learned about Newton's Second Law of motion earlier in this chapter. Newton's Second Law describes the relationship between the motion of an object and the net force on the object. We said that the motion of an object, and therefore its momentum, can only change when a resultant force is acting on it. We can therefore say that because a net force causes an object to move, it also causes its momentum to change. We can now define Newton's Second Law of motion in terms of momentum.

Definition: Newton's Second Law of Motion (N2)
The net or resultant force acting on an object is equal to the rate of change of momentum.

Mathematically, Newton's Second Law can be stated as:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

12.5.6 Impulse

Impulse is the product of the net force and the time interval for which the force acts. Impulse is defined as:

$$\text{Impulse} = F \cdot \Delta t \tag{12.8}$$

However, from Newton's Second Law, we know that

$$\begin{aligned} F &= \frac{\Delta p}{\Delta t} \\ \therefore \quad F \cdot \Delta t &= \Delta p \\ &= \text{Impulse} \end{aligned}$$

Therefore,

$$\text{Impulse} = \Delta p$$

Impulse is equal to the change in momentum of an object. From this equation we see, that for a given change in momentum, $F_{net}\Delta t$ is fixed. Thus, if F_{net} is reduced, Δt must be increased (i.e. a smaller resultant force must be applied for longer to bring about the same change in momentum). Alternatively if Δt is reduced (i.e. the resultant force is applied for a shorter period) then the resultant force must be increased to bring about the same change in momentum.

Worked Example 96: Impulse and Change in momentum

Question: A 150 N resultant force acts on a 300 kg trailer. Calculate how long it takes this force to change the trailer's velocity from 2 m·s^{-1}to 6 m·s^{-1}in the same direction. Assume that the forces acts to the right.

Answer

Step 1 : Identify what information is given and what is asked for

The question explicitly gives

- the trailer's mass as 300 kg,
- the trailer's initial velocity as 2 m·s^{-1}to the right,
- the trailer's final velocity as 6 m·s^{-1}to the right, and
- the resultant force acting on the object

all in the correct units!

We are asked to calculate the time taken Δt to accelerate the trailer from the 2 to 6 m·s^{-1}. From the Law of Momentum,

$$\begin{aligned} F_{net}\Delta t &= \Delta p \\ &= mv_f - mv_i \\ &= m(v_f - v_i). \end{aligned}$$

Thus we have everything we need to find Δt!

Step 2 : Choose a frame of reference

Choose right as the positive direction.

Step 3 : Do the calculation and quote the final answer

$$\begin{aligned} F_{net}\Delta t &= m(v_f - v_i) \\ (+150\,\mathrm{N})\Delta t &= (300\,\mathrm{kg})((+6\,\mathrm{m\cdot s^{-1}}) - (+2\,\mathrm{m\cdot s^{-1}})) \\ (+150\,\mathrm{N})\Delta t &= (300\,\mathrm{kg})(+4\,\mathrm{m\cdot s^{-1}}) \\ \Delta t &= \frac{(300\,\mathrm{kg})(+4\,\mathrm{m\cdot s^{-1}})}{+150\,\mathrm{N}} \\ \Delta t &= 8\,\mathrm{s} \end{aligned}$$

It takes 8 s for the force to change the object's velocity from 2 m·s^{-1}to the right to 6 m·s^{-1}to the right.

Worked Example 97: Impulsive cricketers!

Question: A cricket ball weighing 156 g is moving at 54 km·hr^{-1}towards a batsman. It is hit by the batsman back towards the bowler at 36 km·hr^{-1}. Calculate

1. the ball's impulse, and
2. the average force exerted by the bat if the ball is in contact with the bat for 0,13 s.

Answer

Step 1 : Identify what information is given and what is asked for

The question explicitly gives

- the ball's mass,
- the ball's initial velocity,
- the ball's final velocity, and

- the time of contact between bat and ball

We are asked to calculate the impulse

$$\text{Impulse} = \Delta p = F_{net}\Delta t$$

Since we do not have the force exerted by the bat on the ball (F_{net}), we have to calculate the impulse from the change in momentum of the ball. Now, since

$$\begin{aligned}\Delta p &= p_f - p_i \\ &= mv_f - mv_i,\end{aligned}$$

we need the ball's mass, initial velocity and final velocity, which we are given.

Step 2 : Convert to S.I. units

Firstly let us change units for the mass

$$\begin{aligned}1000\text{ g} &= 1\,\text{kg} \\ \text{So, } 1\text{ g} &= \frac{1}{1000}\text{ kg} \\ \therefore 156 \times 1\text{ g} &= 156 \times \frac{1}{1000}\text{ kg} \\ &= 0{,}156\text{ kg}\end{aligned}$$

Next we change units for the velocity

$$\begin{aligned}1\text{ km}\cdot\text{h}^{-1} &= \frac{1000\,\text{m}}{3\ 600\,\text{s}} \\ \therefore 54 \times 1\text{ km}\cdot\text{h}^{-1} &= 54 \times \frac{1\,000\,\text{m}}{3\,600\,\text{s}} \\ &= 15\,\text{m}\cdot\text{s}^{-1}\end{aligned}$$

Similarly, 36 km·hr^{-1}= 10 m·s^{-1}.

Step 3 : Choose a frame of reference

Let us choose the direction from the batsman to the bowler as the positive direction. Then the initial velocity of the ball is v_i = -15 m·s^{-1}, while the final velocity of the ball is v_f = 10 m·s^{-1}.

Step 4 : Calculate the momentum

Now we calculate the change in momentum,

$$\begin{aligned}p &= p_f - p_i \\ &= mv_f - mv_i \\ &= m(v_f - v_i) \\ &= (0{,}156\,\text{kg})((+10\,\text{m}\cdot\text{s}^{-1}) - (-15\,\text{m}\cdot\text{s}^{-1})) \\ &= +3{,}9\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1} \\ &= 3{,}9\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\text{in the direction from batsman to bowler}\end{aligned}$$

Step 5 : Determine the impulse

Finally since impulse is just the change in momentum of the ball,

$$\begin{aligned}\text{Impulse} &= \Delta p \\ &= 3{,}9\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\text{in the direction from batsman to bowler}\end{aligned}$$

Step 6 : Determine the average force exerted by the bat

$$\text{Impulse} = F_{net}\Delta t = \Delta p$$

We are given Δt and we have calculated the impulse of the ball.

$$\begin{aligned} F_{net}\Delta t &= \text{Impulse} \\ F_{net}(0{,}13\,\text{s}) &= +3{,}9\,\text{N}\cdot\text{s} \\ F_{net} &= \frac{+3{,}9\,\text{N}\cdot\text{s}}{0{,}13\,\text{s}} \\ &= +30\,\text{N} \\ &= 30\,\text{N} \quad \text{in the direction from batsman to bowler} \end{aligned}$$

12.5.7 Exercise

1. Which one of the following is NOT a unit of impulse?

   A $N \cdot s$
   B $kg \cdot \text{m} \cdot \text{s}^{-1}$
   C $J \cdot \text{m} \cdot \text{s}^{-1}$
   D $J \cdot m^{-1} \cdot s$

2. A toy car of mass 1 kg moves eastwards with a speed of 2 m·s^{-1}. It collides head-on with a toy train. The train has a mass of 2 kg and is moving at a speed of 1,5 m·s^{-1}westwards. The car rebounds (bounces back) at 3,4 m·s^{-1}and the train rebounds at 1,2 m·s^{-1}.

   (a) Calculate the change in momentum for each toy.
   (b) Determine the impulse for each toy.
   (c) Determine the duration of the collision if the magnitude of the force exerted by each toy is 8 N.

3. A bullet of mass 20 g strikes a target at 300 m·s^{-1}and exits at 200 m·s^{-1}. The tip of the bullet takes 0,0001s to pass through the target. Determine:

   (a) the change of momentum of the bullet.
   (b) the impulse of the bullet.
   (c) the magnitude of the force experienced by the bullet.

4. A bullet of mass 20 g strikes a target at 300 m·s^{-1}. Determine under which circumstances the bullet experiences the greatest change in momentum, and hence impulse:

   (a) When the bullet exits the target at 200 m·s^{-1}.
   (b) When the bullet stops in the target.
   (c) When the bullet rebounds at 200 m·s^{-1}.

5. A ball with a mass of 200 g strikes a wall at right angles at a velocity of 12 m·s^{-1}and rebounds at a velocity of 9 m·s^{-1}.

   (a) Calculate the change in the momentum of the ball.
   (b) What is the impulse of the wall on the ball?
   (c) Calculate the magnitude of the force exerted by the wall on the ball if the collision takes 0,02s.

6. If the ball in the previous problem is replaced with a piece of clay of 200 g which is thrown against the wall with the same velocity, but then sticks to the wall, calculate:

   (a) The impulse of the clay on the wall.
   (b) The force exerted by the clay on the wall if it is in contact with the wall for 0,5 s before it comes to rest.

12.5.8 Conservation of Momentum

In the absence of an external force acting on a system, momentum is conserved.

Definition: Conservation of Linear Momentum
The total linear momentum of an isolated system is constant. An isolated system has no forces acting on it from the outside.

This means that in an isolated system the total momentum before a collision or explosion is equal to the total momentum after the collision or explosion.

Consider a simple collision of two billiard balls. The balls are rolling on a frictionless surface and the system is isolated. So, we can apply conservation of momentum. The first ball has a mass m_1 and an initial velocity v_{i1}. The second ball has a mass m_2 and moves towards the first ball with an initial velocity v_{i2}. This situation is shown in Figure 12.14.

Figure 12.14: Before the collision.

The total momentum of the system before the collision, p_i is:

$$p_i = m_1 v_{i1} + m_2 v_{i2}$$

After the two balls collide and move away they each have a different momentum. If the first ball has a final velocity of v_{f1} and the second ball has a final velocity of v_{f2} then we have the situation shown in Figure 12.15.

Figure 12.15: After the collision.

The total momentum of the system after the collision, p_f is:

$$p_f = m_1 v_{f1} + m_2 v_{f2}$$

This system of two balls is isolated since there are no external forces acting on the balls. Therefore, by the principle of conservation of linear momentum, the total momentum before the collision is equal to the total momentum after the collision. This gives the equation for the conservation of momentum in a collision of two objects,

$$p_i = p_f$$
$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

m_1 : mass of object 1 (kg)
m_2 : mass of object 2 (kg)
v_{i1} : initial velocity of object 1 (m·s^{-1}+ direction)
v_{i2} : initial velocity of object 2 (m·s^{-1}- direction)
v_{f1} : final velocity of object 1 (m·s^{-1}- direction)
v_{f2} : final velocity of object 2 (m·s^{-1}+ direction)

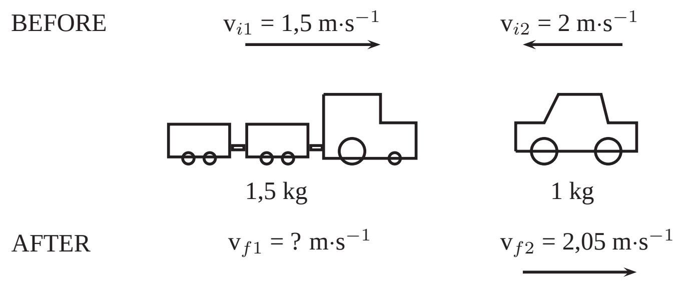
This equation is always true - momentum is always conserved in collisions.

Worked Example 98: Conservation of Momentum 1

Question: A toy car of mass 1 kg moves westwards with a speed of 2 m·s^{-1}. It collides head-on with a toy train. The train has a mass of 1,5 kg and is moving at a speed of 1,5 m·s^{-1}eastwards. If the car rebounds at 2,05 m·s^{-1}, calculate the velocity of the train.

Answer

Step 1 : Draw rough sketch of the situation



Step 2 : Choose a frame of reference

We will choose to the east as positive.

Step 3 : Apply the Law of Conservation of momentum

$$\begin{aligned}
p_i &= p_f \\
m_1 v_{i1} + m_2 v_{i2} &= m_1 v_{f1} + m_2 v_{f2} \\
(1{,}5\,\mathrm{kg})(+1{,}5\,\mathrm{m\cdot s^{-1}}) + (2\,\mathrm{kg})(-2\,\mathrm{m\cdot s^{-1}}) &= (1{,}5\,\mathrm{kg})(v_{f1}) + (2\,\mathrm{kg})(2{,}05\,\mathrm{m\cdot s^{-1}}) \\
2{,}25\,\mathrm{kg\cdot\ m\cdot s^{-1}} - 4\,\mathrm{kg\cdot\ m\cdot s^{-1}} - 4{,}1\,\mathrm{kg\cdot\ m\cdot s^{-1}} &= (1{,}5\,\mathrm{kg})\ v_{f1} \\
5{,}85\,\mathrm{kg\cdot\ m\cdot s^{-1}} &= (1{,}5\,\mathrm{kg})\ v_{f1} \\
v_{f1} &= 3{,}9\,\mathrm{m\cdot s^{-1}}\text{eastwards}
\end{aligned}$$

Worked Example 99: Conservation of Momentum 2

Question: A helicopter flies at a speed of 275 m·s^{-1}. The pilot fires a missile forward out of a gun barrel at a speed of 700 m·s^{-1}. The respective masses of the helicopter and the missile are 5000 kg and 50 kg. Calculate the new speed of the helicopter immmediately after the missile had been fired.

Answer

Step 1 : Draw rough sketch of the situation

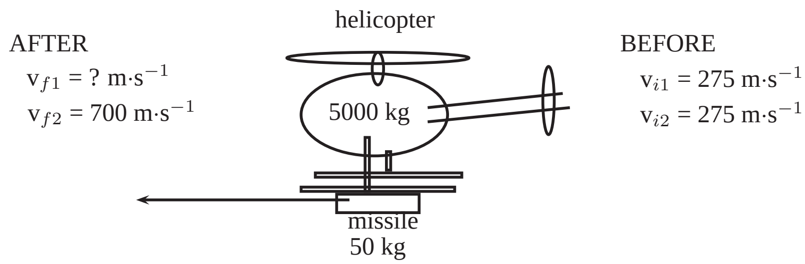


Figure 12.16: helicopter and missile

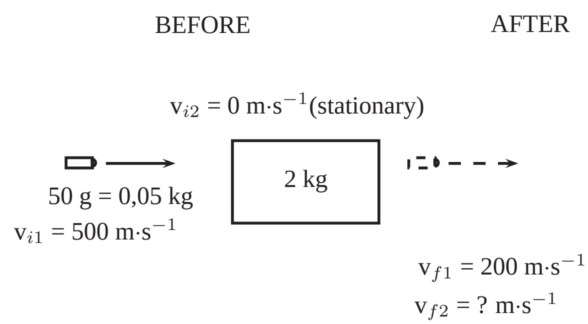
Step 2 : Analyse the question and list what is given
$m_1 = $ 5000 kg
$m_2 = $ 50 kg
$v_{i1} = v_{i2} = $ 275 m·s^{-1}
$v_{f1} = $?
$v_{f2} = $ 700 m·s^{-1}

Step 3 : Apply the Law of Conservation of momentum
The helicopter and missile are connected initially and move at the same velocity. We will therefore combine their masses and change the momentum equation as follows:

$$\begin{aligned}
p_i &= p_f \\
(m_1 + m_2)v_i &= m_1 v_{f1} + m_2 v_{f2} \\
(5000\,\mathrm{kg} + 50\,\mathrm{kg})(275\,\mathrm{m\cdot s^{-1}}) &= (5000\,\mathrm{kg})(v_{f1}) + (50\,\mathrm{kg})(700\,\mathrm{m\cdot s^{-1}}) \\
1388750\,\mathrm{kg\cdot\ m\cdot s^{-1}} - 35000\,\mathrm{kg\cdot\ m\cdot s^{-1}} &= (5000\,\mathrm{kg})(v_{f1}) \\
v_{f1} &= 270{,}75\,\mathrm{m\cdot s^{-1}}
\end{aligned}$$

Note that speed is asked and not velocity, therefore no direction is included in the answer.

Worked Example 100: Conservation of Momentum 3
Question: A bullet of mass 50 g travelling horizontally at 500 m·s^{-1}strikes a stationary wooden block of mass 2 kg resting on a smooth horizontal surface. The bullet goes through the block and comes out on the other side at 200 m·s^{-1}. Calculate the speed of the block after the bullet has come out the other side.
Answer
Step 1 : Draw rough sketch of the situation



Step 2 : Choose a frame of reference
We will choose to the right as positive.

Step 3 : Apply the Law of Conservation of momentum

$$\begin{aligned}
p_i &= p_f \\
m_1 v_{i1} + m_2 v_{i2} &= m_1 v_{f1} + m_2 v_{f2} \\
(0{,}05\,\text{kg})(+500\,\text{m}\cdot\text{s}^{-1}) + (2\,\text{kg})(0\,\text{m}\cdot\text{s}^{-1}) &= (0{,}05\,\text{kg})(+200\,\text{m}\cdot\text{s}^{-1}) + (2\,\text{kg})(v_{f2}) \\
25 + 0 - 10 &= 2\ v_{f2} \\
v_{f2} &= 7{,}5\,\text{m}\cdot\text{s}^{-1}\text{in the same direction as the bullet}
\end{aligned}$$

12.5.9 Physics in Action: Impulse

A very important application of impulse is improving safety and reducing injuries. In many cases, an object needs to be brought to rest from a certain initial velocity. This means there is a certain specified change in momentum. If the time during which the momentum changes can be increased then the force that must be applied will be less and so it will cause less damage. This is the principle behind arrestor beds for trucks, airbags, and bending your knees when you jump off a chair and land on the ground.

Air-Bags in Motor Vehicles

Air bags are used in motor vehicles because they are able to reduce the effect of the force experienced by a person during an accident. Air bags extend the time required to stop the momentum of the driver and passenger. During a collision, the motion of the driver and passenger carries them towards the windshield. If they are stopped by a collision with the windshield, it would result in a large force exerted over a short time in order to bring them to a stop. If instead of hitting the windshield, the driver and passenger hit an air bag, then the time of the impact is increased. Increasing the time of the impact results in a decrease in the force.

Padding as Protection During Sports

The same principle explains why wicket keepers in cricket use padded gloves or why there are padded mats in gymnastics. In cricket, when the wicket keeper catches the ball, the padding is slightly compressible, thus reducing the effect of the force on the wicket keepers hands. Similarly, if a gymnast falls, the padding compresses and reduces the effect of the force on the gymnast's body.

Arrestor Beds for Trucks

An arrestor bed is a patch of ground that is softer than the road. Trucks use these when they have to make an emergency stop. When a trucks reaches an arrestor bed the time interval over which the momentum is changed is increased. This decreases the force and causes the truck to slow down.

Follow-Through in Sports

In sports where rackets and bats are used, like tennis, cricket, squash, badminton and baseball, the hitter is often encouraged to follow-through when striking the ball. High speed films of the collisions between bats/rackets and balls have shown that following through increases the time over which the collision between the racket/bat and ball occurs. This increase in the time of the collision causes an increase in the velocity change of the ball. This means that a hitter can cause the ball to leave the racket/bat faster by following through. In these sports, returning the ball with a higher velocity often increases the chances of success.

Crumple Zones in Cars

Another safety application of trying to reduce the force experienced is in crumple zones in cars. When two cars have a collision, two things can happen:

1. the cars bounce off each other, or
2. the cars crumple together.

Which situation is more dangerous for the occupants of the cars? When cars bounce off each other, or rebound, there is a larger change in momentum and therefore a larger impulse. A larger impulse means that a greater force is experienced by the occupants of the cars. When cars crumple together, there is a smaller change in momentum and therefore a smaller impulse. The smaller impulse means that the occupants of the cars experience a smaller force. Car manufacturers use this idea and design crumple zones into cars, such that the car has a greater chance of crumpling than rebounding in a collision. Also, when the car crumples, the change in the car's momentum happens over a longer time. Both these effects result in a smaller force on the occupants of the car, thereby increasing their chances of survival.

Activity :: Egg Throw : This activity demonstrates the effect of impulse and how it is used to improve safety. Have two learners hold up a bed sheet or large piece of fabric. Then toss an egg at the sheet. The egg should not break, because the collision between the egg and the bed sheet lasts over an extended period of time since the bed sheet has some give in it. By increasing the time of the collision, the force of the impact is minimized. Take care to aim at the sheet, because if you miss the sheet, you will definitely break the egg and have to clean up the mess!

12.5.10 Exercise

1. A canon, mass 500 kg, fires a shell, mass 1 kg, horizontally to the right at 500 m·s^{-1}. What is the magnitude and direction of the initial recoil velocity of the canon?
2. A trolley of mass 1 kg is moving with a speed of 3 m·s^{-1}. A block of wood, mass 0,5 kg, is dropped vertically into the trolley. Immediately after the collision, the speed of the trolley and block is 2 m·s^{-1}. By way of calculation, show whether momentum is conserved in the collision.

3. A 7200 kg empty railway truck is stationary. A fertilizer firm loads 10800 kg fertilizer into the truck. A second, identical, empty truck is moving at 10 m·s^{-1}when it collides with the loaded truck.

   (a) If the empty truck stops completely immediately after the collision, use a conservation law to calculate the velocity of the loaded truck immediately after the collision.

   (b) Calculate the distance that the loaded truck moves after collision, if a constant frictional force of 24 kN acts on the truck.

4. A child drops a squash ball of mass 0,05 kg. The ball strikes the ground with a velocity of 4 m·s^{-1}and rebounds with a velocity of 3 m·s^{-1}. Does the law of conservation of momentum apply to this situation? Explain.

5. A bullet of mass 50 g travelling horizontally at 600 m·s^{-1}strikes a stationary wooden block of mass 2 kg resting on a smooth horizontal surface. The bullet gets stuck in the block.

   (a) Name and state the principle which can be applied to find the speed of the block-and-bullet system after the bullet entered the block.

   (b) Calculate the speed of the bullet-and-block system immediately after impact.

   (c) If the time of impact was 5 x 10^{-4} seconds, calculate the force that the bullet exerts on the block during impact.

12.6 Torque and Levers

12.6.1 Torque

This chapter has dealt with forces and how they lead to motion in a straight line. In this section, we examine how forces lead to rotational motion.

When an object is fixed or supported at one point and a force acts on it a distance away from the support, it tends to make the object turn. The moment of force or *torque* (symbol, τ read *tau*) is defined as the product of the distance from the support or pivot (r) and the component of force perpendicular to the object, $F_\perp$.

$$\tau = F_\perp \cdot r \tag{12.9}$$

Torque can be seen as a rotational force. The unit of torque is N·m and torque is a vector quantity. Some examples of where torque arises are shown in Figures 12.17, 12.18 and 12.19.

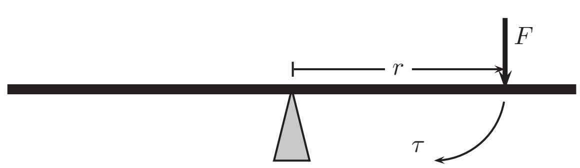


Figure 12.17: The force exerted on one side of a see-saw causes it to swing.

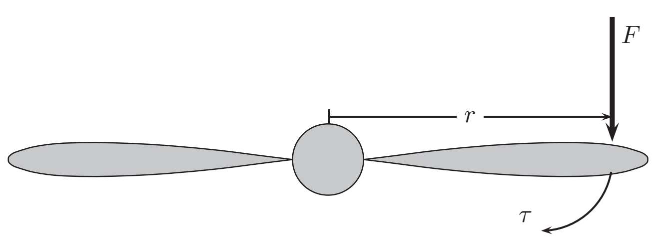


Figure 12.18: The force exerted on the edge of a propellor causes the propellor to spin.